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Some Effects of Variation of Control Law Parameters in the Simulated ET 316 System

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7 ~~SOME EFFECTS OF VARIATION OF CONTROL LAW PARAMETERS~~

~~IN THE SIMULATED ET 316 SYSTEM~~ [C]. 8

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Study: R20

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86/Research/1608

APPROVED: Thomas W. Cannick

Director

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APRE RESEARCH MEMORANDUM N/1

SOME EFFECTS OF VARIATION OF CONTROL LAW PARAMETERS IN THE SIMULATED ET 316 SYSTEM

by

L.R. Speight

SUMMARY

In the ET 316 system the operator is required to track targets visually via a servo-assisted sight. It seems clear that the choice of control law, relating the operator's joystick deflection to the tracker output, will have a considerable influence on tracking error.

An experiment has been conducted wherein control law parameters have been varied for three different simulated target courses. Two of the targets were straight and level fliers but with different crossing distances, and the remaining target had an added 2g vertical weave. Eleven RA personnel were employed as subjects.

The results have been presented as a series of approximate functional relationships between tracking accuracy or acquisition and settling time on the one hand, and control law parameters on the other. It has been concluded that:

(a) The relationship between tracking accuracy or acquisition and settling time and control law parameters is dependent on the particular target course, and so there is unlikely to be a unique optimum law suitable for all targets.

(b) For the targets used in the experiment, a control law with parameters roughly as follows appears to be a reasonable compromise:

position	1.1 deg.
velocity	4.8 deg/sec.
acceleration	2.2 deg/sec^2.

CONTENTS

SOME EFFECTS OF VARIATION OF CONTROL LAW PARAMETERS IN THE SIMULATED ET 316 SYSTEM

by

L.R. Speight

INTRODUCTION

Background

1. In the ET 316 system the operator is required to track targets visually via a servo-assisted sight. Quite obviously the choice of control law, relating the operator's joystick deflection to the tracker output, will have a considerable influence on the expected tracking error. Accordingly, this aspect of design has been studied experimentally both by the Royal Radar Establishment (RRE), Malvern, and by the British Aircraft Corporation (BAC), Stevenage, although the lines of approach used by these two agencies have been rather different.

2. In a preliminary experiment[1] RRE compared a pure velocity control with an acceleration control (the latter with a small position component added), using as a criterion of tracking accuracy the integrated root mean square (r.m.s.) error over a defined portion of the run. Although differences were small, the velocity control seemed to be slightly superior. However, on theoretical grounds Searle[6] had argued that a "mixed" law with position, velocity and acceleration terms in the ratio 1:4:8 should be optimal for tracking. An experiment comparing a law of this kind with velocity controls with different amounts of position aiding confirmed the superiority of the "mixed" law[2]. This series of investigations thus led to a preferred law such that full deflection of the joystick control gave a demand of:

position	0.5 deg.
velocity	2 deg/sec.
acceleration	4 deg/sec^2.

and this was used by AORE (now APRE) when they investigated the effects of different types of control[7].

3. The BAC approach was more empirical. A number of velocity controls with different amounts of position aiding were examined to select one which gave good results over the major part of the

missile coverage[4]. It was concluded that the best law to achieve this aim was one that for full joystick deflection gave:

position	1.43 deg.	(0.025 rad)
velocity	4.30 deg/sec.	(0.075 rad/sec.)

However, this law could not generate rates sufficient to track some classes of engageable target. A satisfactory means of producing this additional capability, without prejudicing performance over the main portion of the missile coverage, was to add a certain amount of acceleration control. This series of experiments thus led to a final law (as given in Reference 5) such that full joystick deflection gave:

position	1.53 deg.	(0.0267 rad)
velocity	4.58 deg/sec.	(0.08 rad/sec.)
acceleration	2.29 deg/sec^2.	(0.04 rad/sec^2.)

4. The present experiment was devised to complement these previous investigations. It aimed at expressing tracking errors as an approximate mathematical function of control law parameters, at least over a limited range of these latter and for a limited number of target courses. As well as describing the results in a general way, it was hoped that the "response surfaces" so fitted to the data might indicate optimal values of control law constants. It was also planned to investigate the related problem of acquisition and settling time in similar fashion during the same experiment.

Aim

5. To examine the effect of variations of control law constants on tracking accuracy and on acquisition and settling time in the simulated ET 316 system.

METHOD

General

6. In the type of control law considered there are three constants - the position, velocity and acceleration components - which in theory can be varied independently. In practice, however, there are some combinations of velocity and acceleration which cannot be used in conjunction: if they are together too high in value, the system is too sensitive for accurate tracking, and if together they are too low, the system becomes incapable of following those targets with the fastest rates. This suggests that for the practical purposes of this investigation these two parameters can be combined into a single "velocity vs acceleration" dimension, at one extreme of which would be pure velocity control, and at the other pure acceleration control, with different admixtures of these two in between. In the following experiment this dimension has been defined by the line

$$V + 2A = 10$$

where V is the value of the velocity component in deg/sec.

and A is the value of the acceleration component in deg/sec^2.

7. In the general experimental method used it is assumed that a functional relationship exists between the magnitude of tracking errors and control law parameters. The form of this relationship is unknown, but it is further assumed that over a sufficiently small range it can be approximated by a polynomial of the type

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 \ldots (1)$$

where η is the expected magnitude of tracking error

x_1 is the value of the position component expressed on a suitable scale

x_2 is the value of the velocity-acceleration component expressed on a suitable scale

and β_0, β_1, etc. are the lower-order terms in a Taylor series expansion of the "true" function.

Estimates b_0, b_1, etc. of the parameters β_0, β_1, etc. may be obtained by experimentation, using one of a class of experimental designs which have been evolved for this purpose. A full description of the design used in this experiment is given in Appendix B.

<u>Pilot Experiments</u>

8. As a lower-order polynomial is likely to be an adequate model over a limited range only, and as in any case our interest is confined to that region where tracking optima are likely to lie, some small scale pilot experiments were carried out in order to:

(a) Determine the range of values over which the main experiment should be conducted.

(b) Suggest suitable scales in terms of which the control law parameters should be expressed.

(c) Throw some light on other effects, such as transfer of training, day-to-day variation, learning and within-block trends, which might have some bearing on the conduct and design of the main experiment.

Although it would have been preferable to use properly representative subjects for the pilot experiments, the supply of Army personnel available for such purposes is strictly limited, and two scientific staff acted in this capacity.

9. The experiments were concerned with tracking accuracy only and are described briefly in Appendix A, and the implications they were thought to have for the design of the main experiment are discussed. In particular, it was concluded that a suitable scale for the position component was given by the transformation

$$x_1 = \log_{10} P$$

where P is the value of the position component in degrees. No simple transformation for the velocity vs acceleration dimension

proved adequate, but the following rather artificial scale provided a good fit to the data:

$$x_2 = A/(V + 0.5A), \quad (V + 2A = 10)$$

where V is the value of the velocity component in deg/sec, and A is the value of the acceleration component in deg/sec^2. The experimental variables were therefore defined in this way.

Subjects

10. Eleven personnel from 37 A.D. Regt RA took part in this experiment. This was the same sample of 12 (less one man unavoidably absent on leave) who had participated in a study of skill retention reported in Reference 8. It was shown there that these 12 could be regarded as a representative sample from the potential user population.

11. Prior to the present investigation, these operators had undertaken a minimum of 128 tracking runs, using for the most part a control law with the following constants:

position	1.5 deg.
velocity	4.6 deg/sec.
acceleration	2.4 deg/sec^2

Equipment

12. The experiment was conducted at RRE, Malvern, using the simulation equipment developed there. RRE have provided a fairly full description of this equipment in Appendix A of Reference 7. The operator's station consisted of a padded chair in front of an elbow-level, wooden desk-top, at the far end of which was the display, its centre raised 15½" above the surface. This display consisted of a 14" EMI Mark IV, type TTM6/14A, TV monitor, representing a 4.8° by 3.6° field of view. It was situated roughly 22" from the operator's eyes, although this varied with posture, which was uncontrolled. Electronically generated cross-wires were used, with a small gap at their junction. Both target and cross-wires were dark against a bright background. The control was a miniature free-moving joystick, let into the desk-top. It measured 3 1/16" from pivot to tip and permitted an angular movement of ± 32½°. The sense of this control was such that deflection of the joystick to the left moved the target spot to the left, and a deflection towards the operator's body moved the target spot downwards.

Procedure

13. The targets used in this experiment were all of velocity 1,000 ft/sec. and initial slant range 30,000 ft. Three target courses were used, with crossing distances of:

(a) 2,500 ft.
(b) 5,000 ft.
(c) 10,000 ft.

The first and last of these were straight and level fliers with height zero feet, but course (b) had an added 2g vertical weave (sine wave) with a period of 12 sec. and peak amplitude 234 ft.

14. At the start of each run the target was displaced from the cross-wires 200 ft. vertically and 200 ft. to the right. It was hoped in this way to obtain a rough estimate of the effect of the control law on target acquisition and settling time. Each tracking run lasted 27 sec, tracking errors being integrated over the last two-thirds of this period. The index used for this purpose was r.m.s. radial error measured in feet (although the analysis which follows is conducted in terms of log r.m.s. error, as this transformation is required for statistical validity). Pen records of tracking errors were taken over the first 9 sec. of each run and these showed the miss distance, in feet, in both vertical and horizontal planes. For the purposes of this experiment "acquisition and settling time" was defined as the time from the start of the run until the operator reduced his error in both planes to less than 20 ft, subsequently holding his error within these boundaries for at least 2 sec. Acquisition time was thus determined in each case by measurement on the appropriate pen record.

15. Each of the 11 operators undertook 11 blocks of tracking runs, each block being with a different control law. The control law parameters used in this experiment are listed below:

TABLE I

Block Number (Control Law)	Control Law Parameters				
	Position (deg.)	Velocity (deg/sec)	Acceleration (deg/sec^2)	Transformed Scale	
				x_1	x_2
1, 2, 3	1.8	4.4	2.8	0.25527	0.48276
4	1.2	5.2	2.4	0.07918	0.37500
5	1.2	3.737	3.131	0.07918	0.59052
6	2.7	5.2	2.4	0.43136	0.37500
7	2.7	3.737	3.131	0.43136	0.59052
8	1.015	4.4	2.8	0.00625	0.48276
9	3.194	4.4	2.8	0.50430	0.48276
10	1.8	5.582	2.209	0.25527	0.33037
11	1.8	3.495	3.253	0.25527	0.63515

These blocks were arranged in a Latin Square design to control any long-term trends, such as learning effects. Each block consisted of 7 runs, the first three being of target course (c), the next two of course (a), and the remaining two of course (b), defined in Paragraph 13. The scores from the first run in each block were excluded from the analysis, in order to minimise short-term transfer effects. The scored pair of runs for each target course consisted of a left and a right crossing target, presented in random order.

RESULTS

16. The statistical analysis of the data is given in Appendix B. The analysis yielded fitted equations of the following type:

$$\hat{y} = b_o + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{22}x_2^2 + b_{12}x_1x_2 \quad \ldots\ldots (2)$$

where

$\hat{y}$ is the estimated response (log r.m.s. error for tracking accuracy or time in seconds for acquisition and settling)

x_1 is the position component on the transformed scale

x_2 is the velocity vs acceleration component on the transformed scale

and b_o, b_1, etc. are estimates of the constants β_o, β_1, etc. in Equation (1).

It was found that the data demanded separate equations for each target course. The values of the fitted constants are given in Table II.

TABLE II

Target Course (Crossing Distance)	Fitted Constants					
	b_o	b_1	b_2	b_{11}	b_{22}	b_{12}
Tracking Accuracy						
10,000 ft	0.4815	0.2570	0.2990	0.6952	0.1081	-1.3077
2,500 ft	0.6057	-0.3579	0.5655	0.5439	-0.7377	0.4538
5,000 ft + weave	0.9224	-0.1109	-0.1247	0.7774	-0.0261	0.3733
Acquisition Time						
10,000 ft	0.9613	-6.0827	4.9437	12.1626	-2.2141	7.3660
2,500 ft	0.8040	-0.6023	3.0972	5.5201	-0.8197	-2.8446
5,000 ft + weave	1.1579	-7.4643	3.9556	5.4423	-3.8654	16.2292

17. By mere inspection of the numerical values of these sets of coefficients it is difficult to visualise the surfaces which they represent. They can be illustrated graphically, however, by drawing a system of contours of equal response in the space of x_1 and x_2. In this way a kind of map of tracking accuracy or acquisition time can be built up, so that the way in which estimated response varies with the values of control law constants may readily be appreciated. Such maps are given in Figures 1-6. The points at which measurements were taken are indicated in these Figures, together with the estimates of response actually obtained at these points. Also shown, in the form of a long dotted line, is the major axis of the fitted system (i.e. the line along which there is least variation of response). In those cases where there is a stationary point within the area covered by the experiment, the minor axis is also indicated (although in Appendix B, Paragraph B.7, a note of caution is sounded on the interpretation of these principal axes). For all three target courses,

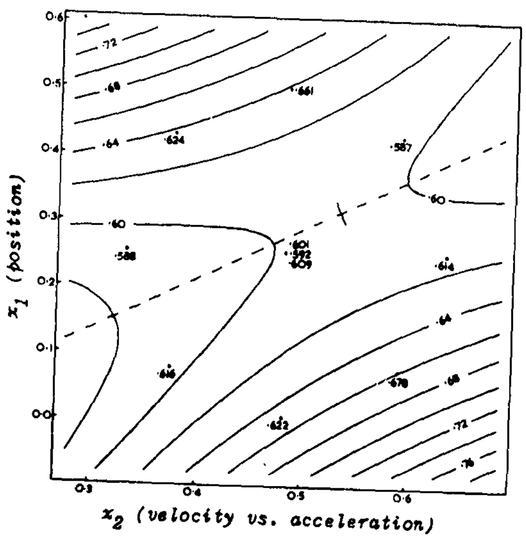

Figure 1

Tracking accuracy. Target crossing distance 10,000 ft. Contours are spaced at intervals of 0.02 log r.m.s. error.

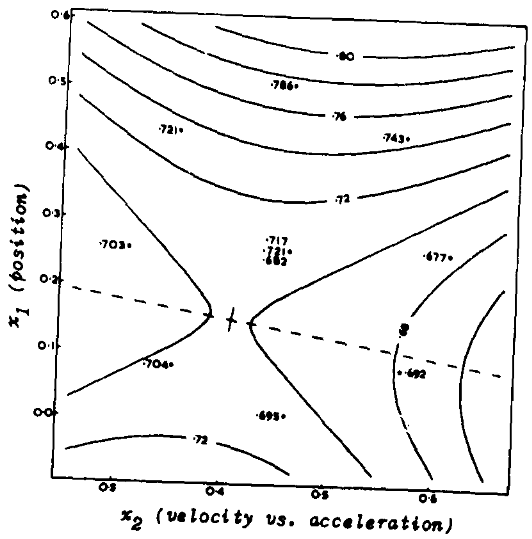

Figure 2

Tracking accuracy. Target crossing distance 2,500 ft. Contours are spaced at intervals of 0.02 log r.m.s. error.

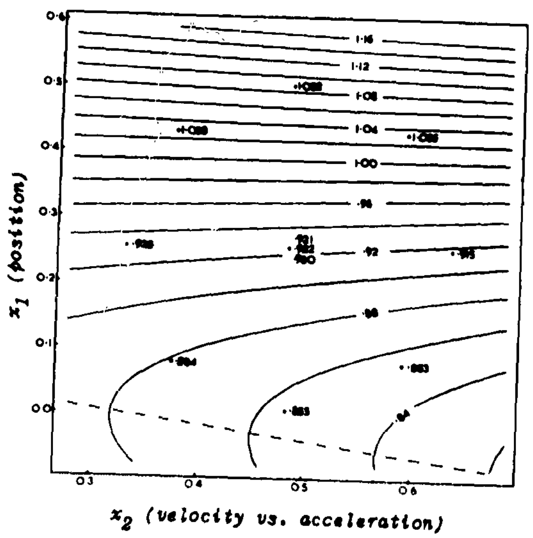

Figure 3

Tracking accuracy. Target crossing distance 5,000 ft. with 2g vertical weave. Contours are spaced at intervals of 0.02 log r.m.s. error.

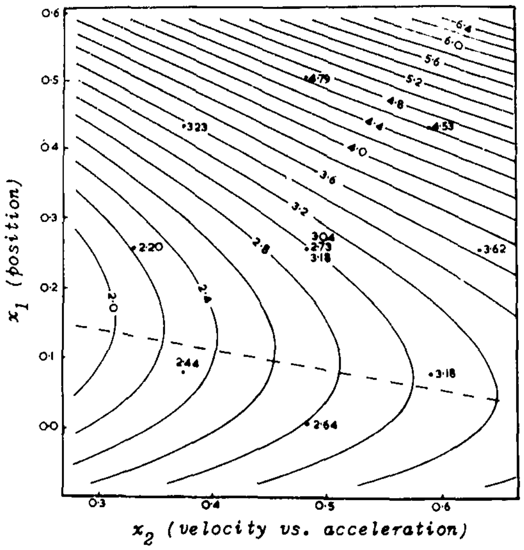

Figure 4

Acquisition and settling time.
Target crossing distance 10,000 ft.
Contours are spaced at intervsls of 0.2 secs.

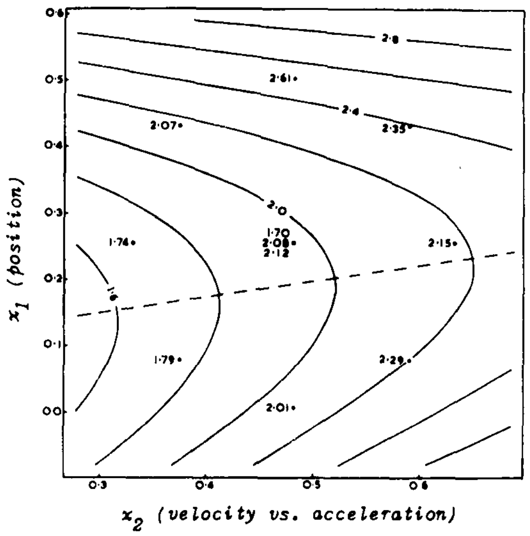

Figure 5

Acquisition and settling time.
Target crossing distance 2,500 ft.
Contours are spaced at intervals of 0.2 secs.

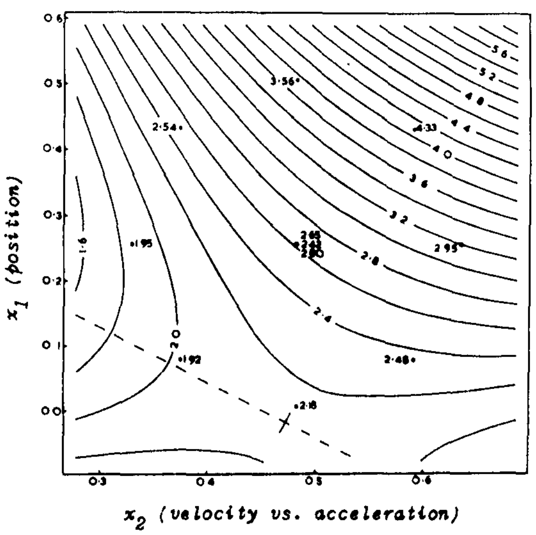

Figure 6

Acquisition and settling time.
Target crossing distance 5,000 ft. with 2g vertical weave. Contours are spaced at intervals of 0.2 secs.

both for tracking accuracy and acquisition time, curvature along the major axis falls far short of statistical significance. Thus the data are not inconsistent with the interpretation that they reflect valley (or inverted ridge) systems, with all curvature taking place in one dimension.

18. It will be understood that, unless the size of the experiment is extremely large, response surfaces will be determined with appreciable experimental error. It would therefore be unwise to place too great a reliance on the finer details of the Figures shown here. This may be seen by considering the width of confidence intervals for estimates of response. Although these will vary for different locations in the experimental region, all points equidistant from the centre of the present design will have intervals of equal size. If, for purposes of illustration, we arbitrarily choose any spot on the outer circumference of experimental points, then the width of 50% and 90% confidence intervals at this location will be as given in Table III.

TABLE III

Target Course (Crossing Distance)	Width of Confidence Interval		Unit of Measurement
	50%	90%	
Tracking Accuracy			
10,000 ft	0.0204	0.0501	log r.m.s. error (ft)
2,500 ft	0.0201	0.0493	
5,000 ft + weave	0.0152	0.0372	
Acquisition Time			
10,000 ft	0.207	0.507	time (sec)
2,500 ft	0.182	0.447	
5,000 ft + weave	0.296	0.725	

19. There is a strong positive relationship between each operator's mean tracking score and his mean acquisition time, taken over all control laws and all target courses in this experiment. The correlation between these two indices was 0.88, which attains a high level of statistical significance. On the other hand, the relationship between intelligence test results (TSG score) and tracking performance was small and very far from attaining statistical significance.

20. From the data of this experiment the population standard deviation for tracking error has been estimated as 0.0838 units. If we can assume that individual tracking abilities approximately follow a log normal distribution, this implies that the difference between operators at the 5th and 95th percentiles is a factor of 1.9. There was no evidence to suggest that this factor differed for different target courses.

DISCUSSION

21. Before considering the results obtained in this experiment, it would perhaps be as well to re-emphasise the difference between BAC's approach to control law optimisation and that adopted here.

It will be recalled that BAC attempted to ensure that error would not be excessive at any point in the missile's coverage; whereas we have attempted to minimize the average error during an engagement, giving full weighting to all segments of the target run within defined limits. Roughly speaking, the former strategy would be most appropriate if it were assumed that missile interception would tend to cluster fairly closely around one range point in future engagements, the location of this point being unknown at present. The latter strategy would be more appropriate if one could assume that the probability of missile interception would be fairly evenly distributed over a wide range band. In fact, our present state of knowledge probably lies somewhere between these two extremes (although possibly nearer the former than the latter) and it is this reasoning which has led us to believe that the two approaches outlined are complementary. It is worth pointing out, however, that the results obtained by the two optimization methods will not necessarily be identical. Of course, if each method leads to similar conclusions, then the confidence in one's final choice of control law would obviously be that much the greater.

22. From Annex A it will be seen that the final pilot experiment showed a clear-cut optimum for the non-manoeuvring, 10,000 ft. crossing distance target, and an inverted ridge system for a 2,500 ft. crossing distance target with 3g vertical weave. Both results were misleading as a guide to the findings of the main experiment, as no true optima were found for straight and level fliers in the latter case, and for the manoeuvring target the inverted ridge was located differently. One factor which might be thought to influence results is that the experimental sample consisted of two groups with different early training histories. Before working with the BAC control law (1.5/4.6/2.4), four of the sample had trained on the RRE law (0.5/2.0/4.0), whereas the remaining seven had trained on the BAC law from the outset. It is possible, then, that early training might have had a long-term residual effect, and that the results, in fact, spring from two separate groups with different optima. As there was no marked learning effect, it was possible to analyse these two sub-samples separately. The two sets of response surfaces so generated proved very similar to each other and to the set based on the pooled data, and so we have discounted early training as a possible explanation. It is felt, however, that the discrepancies between the results of the pilot experiment and those of the main experiment do demonstrate the importance of correct sampling. The fact that the most noticeable difference between samples drawn from different populations is often one of general intelligence or educational attainment should not be allowed to obscure the possibility of differences in less obvious, less easily quantified, but still important associated variables (such as opportunities for exercising other control skills - car driving, for instance - or even such manipulative skills as drawing or writing).

23. Over the range of values covered in the experiment, variations in control law parameters affected tracking accuracy relatively little for the 2,500 ft. crossing distance, rather more for the 10,000 ft. target, but for the weaving target the effect was quite marked. If results had been quoted in terms of r.m.s. errors rather than log r.m.s. errors, this difference would, of course, have appeared even greater. If aircraft are as likely to manoeuvre

as to fly straight and level, then these results suggest that greater weight should be given to the former class of target than to the latter in optimizing the control law. As for acquisition time, as might be expected, control law variations have the greatest effect on targets with the greatest initial rates (i.e. the weaving target, and that with a crossing distance of 10,000 ft).

24. The major axes of the six response surfaces obtained (three for tracking accuracy and three for acquisition time) are reproduced in Figure 7. It will be seen that they come quite close together in the bottom left-hand corner of the diagram, so that a control law represented by the experimental point ringed should be reasonably satisfactory for all three targets used in this experiment. Indeed, if the results for all

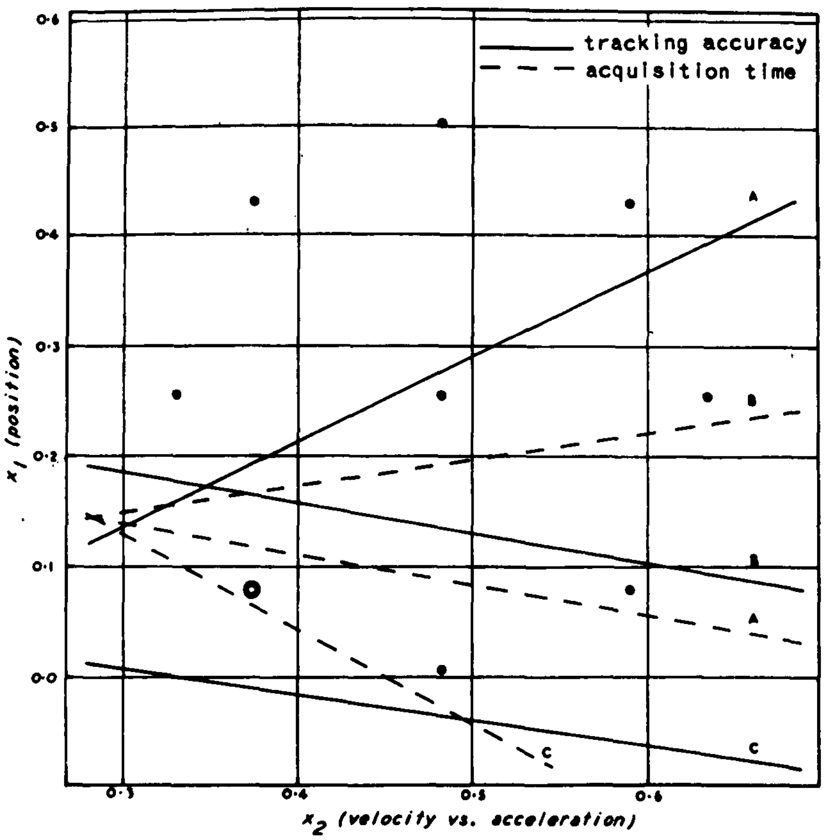

Figure 7: Major Axes of Response Surfaces. Target Courses:
A - 10,000 ft crossing distance
B - 5,000 ft crossing distance
C - 2,500 ft crossing distance with 2g vertical weave

three target courses are pooled, the response surfaces depicted in Figures 8 and 9 result, and consideration of these latter leads to substantially the same choice of control law. (It is unfortunate, however, that the choice of experimental points prompted by the pilot investigation should have proved to be so badly centred for this sample.) The point indicated represents a control law with the following parameters:

position	1.2 deg.
velocity	5.2 deg/sec.
acceleration	2.4 deg/sec^2.

Scaled down to the sensitivity of the BAC law (which latter satisfies the condition $V + 2A = 9.2$), the values of the parameters would read:

position	1.1 deg.
velocity	4.78 deg/sec.
acceleration	2.21 deg/sec^2.

Apart from the rather smaller position term (required for good performance against a manoeuvring target), this is very similar to the law proposed by BAC (1.53/4.58/2.29). However, even discounting the approximate nature of this "optimum", it is obvious that a fair amount of latitude is possible in the neighbourhood of this point without seriously affecting tracking accuracy or acquisition time. There is, in fact, a hint in the data that a control law with an even higher ratio of velocity to acceleration terms would not degrade tracking performance unduly and might aid acquisition, but such a law would lie outside the area covered by the experiment, where the width of confidence interval would be quite large.

25. Turning to the subsidiary results, the strong positive relationship between acquisition performance and tracking accuracy is encouraging. It has been found with Vigilant[3] that performance against short-range targets (which is considered to be mainly an acquisition task) is negatively correlated with performance against long-range targets (which is mainly a tracking task). If this result had been repeated with ET 316, it would have been most inconvenient.

26. In early experiments with simulations of ET 316, a strong positive relationship was found between intelligence and tracking accuracy. A previous report[7] suggested that it might be unwise to accept these results too uncritically, as it was felt that this relationship might disappear with verbal guidance during early learning, and with prolonged training. These latter conditions apply to the present group of operators, and the correlation has indeed dropped to near-zero.

27. Finally, it should be pointed out that the estimate of population standard deviation (0.0838 log units) is obtained on the most highly-trained sample likely to be encountered prior to evaluation.

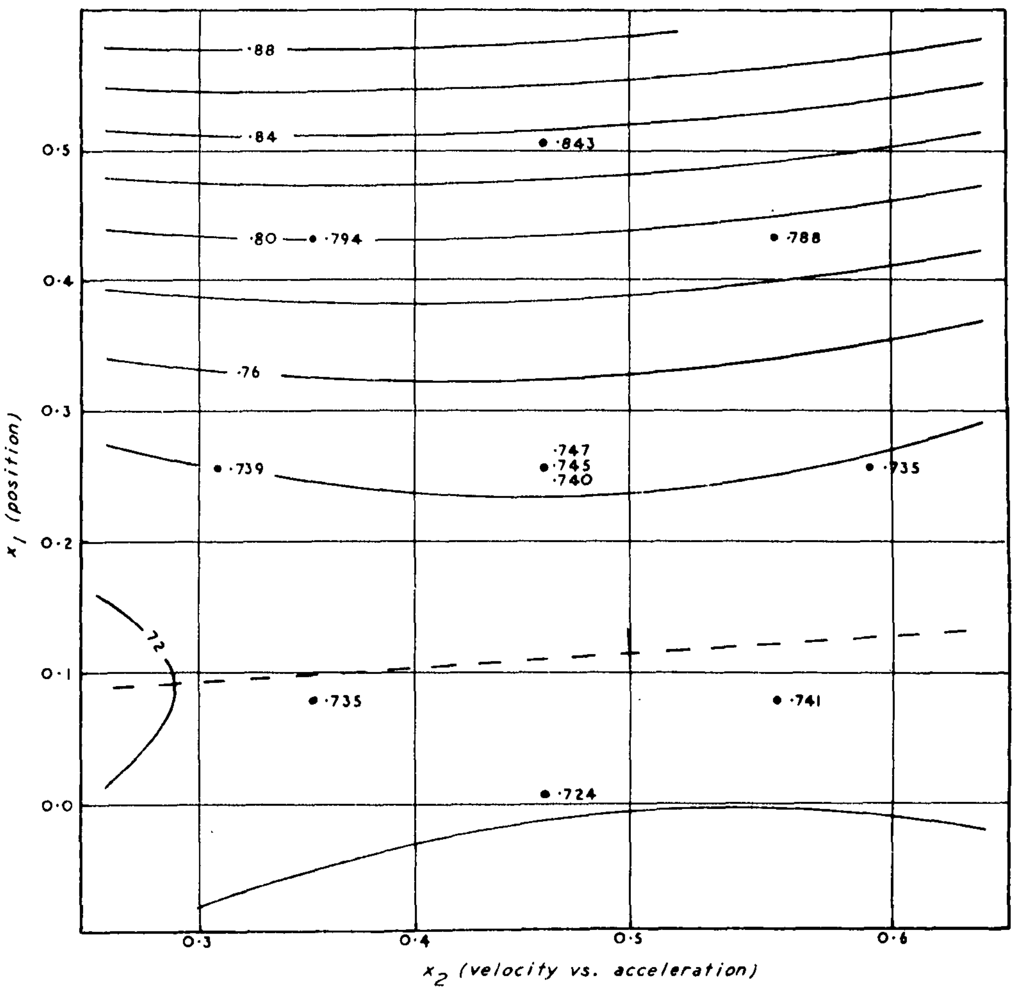

Figure 8 Tracking accuracy. All three target courses pooled. Contours are spaced at intervals of 0.02 log r.m.s. error.

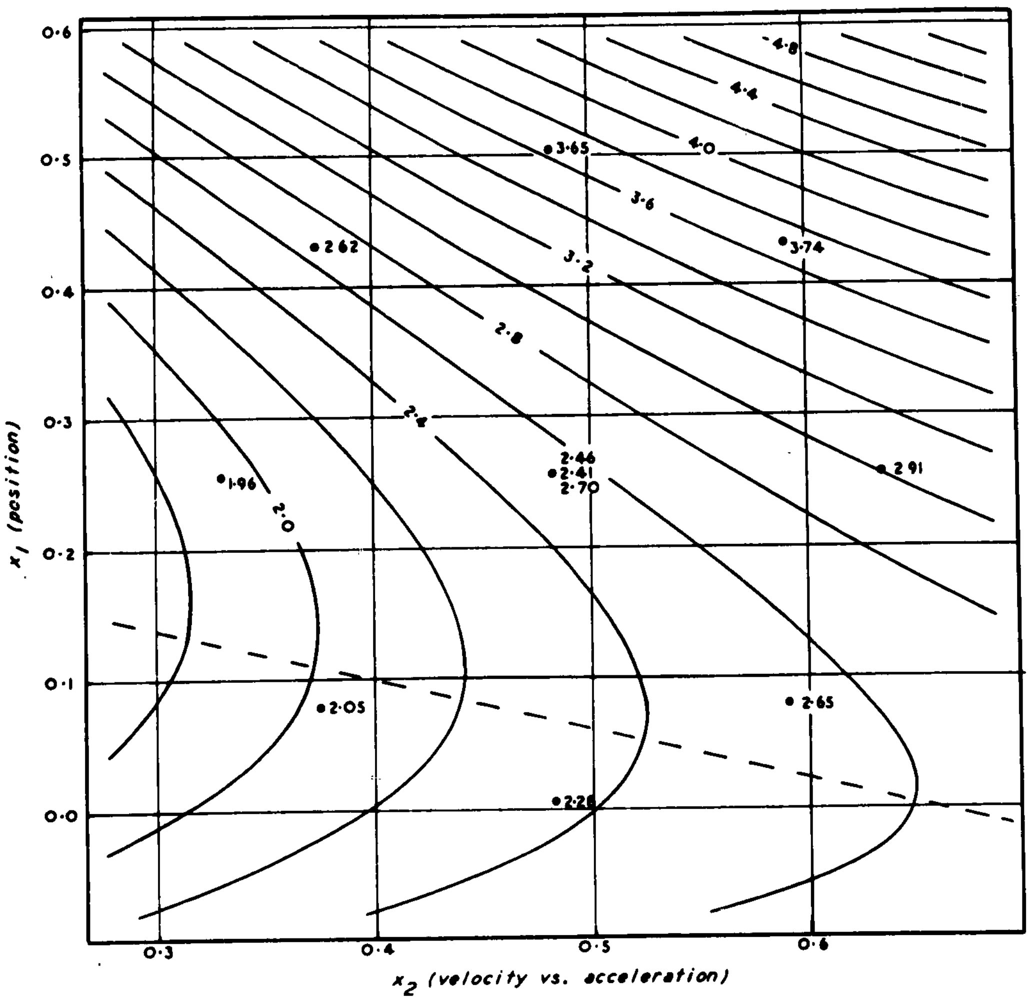

Figure 9 Acquisition and settling time. All three target courses pooled. Contours are spaced at intervals of 0.2 secs.

SUMMARY AND CONCLUSIONS

28. An experiment has been conducted to study some effects of variations of control law parameters in the simulated ET 316 system. The results have been presented as a series of approximate functional relationships between tracking accuracy or acquisition and settling time and control law parameters. It has been concluded that:

(a) The relationship between tracking accuracy or acquisition and settling time and control law parameters is dependent on the particular target course, and so there is unlikely to be a unique optimum law suitable for all targets.

(b) For the targets used in this experiment a control law with parameters roughly as follows appears to be a reasonable compromise:

position	1.1 deg.
velocity	4.8 deg/sec.
acceleration	2.2 deg/sec^2.

ACKNOWLEDGEMENTS

This experiment was conducted at RRE, Malvern, on the equipment developed there, and once more the author wishes to express his thanks, in particular to Messrs. J.S. Bickerdike, G.A. Hawkins and J.H. Hewlett for their help and encouragement.

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ANNEX A

PILOT EXPERIMENTS

General

A.1. The possible range of variations of the three control law parameters is, of course, extremely wide. To carry out a grid of experiments sufficiently widespread to stretch over the whole region of possible operation would be an undertaking too prodigal to contemplate. It is therefore necessary to restrict our interest to a region in which we hope tracking optima will lie and so, as indicated in Paragraph 8 of the main text, a series of experiments was planned in order to:

(a) Determine the range of values over which the main experiment should be conducted.

(b) Suggest suitable scales in terms of which the control law parameters should be expressed.

(c) Throw light on other effects, such as transfer of training, learning, etc.

A.2. Probably the most elegant and economical experimental technique for approaching the region of an optimum is the method of "steepest ascent" pioneered by Box and others (see, for example, Reference A.1). This is essentially a sequential procedure, and its effective use requires analysis between the successive experimental steps. Such an arrangement would have been impractical in the present case, and so the classic "one-factor-at-a-time" approach has been used. Two experiments were conducted varying the velocity and acceleration terms (regarding these as one factor), holding the position term constant. This was followed by one experiment holding velocity and acceleration terms constant and varying the position term. The final experiment was conducted with a grid of points spread out in both dimensions. Detailed analysis is omitted in the following paragraphs, but the experiments and the results obtained are briefly described.

Subjects

A.3. Two members of APRE Scientific Staff took part in Experiments 1 to 3. One of these subjects was omitted for Experiment 4.

Procedure

A.4. In terms of equipment, and in general details of target properties, the procedure used in these experiments were similar to those in the main experiment (see Paragraphs 12-14 of main text). However, errors were integrated over the last 22 sec of each run, allowing only 5 sec for acquisition and settling.

Experiments 1 and 2

A.5. For both these experiments the value of the position term was held constant at 1.6 deg. Measurements were then taken at the following

combinations of velocity and acceleration terms:

(a) 8.4:0.8

(b) 6.8:1.6

(c) 5.2:2.4

(d) 3.6:3.2

(e) 2.0:4.0

($deg/sec:deg/sec^2$). The crossing distances for the three target courses were:

(i) 2,500 ft

(ii) 10,000 ft

(iii) 2,500 ft, but with an added 3 g vertical weave.

A.6. In Experiment 1 each subject undertook 9 blocks of 15 runs each. Each block was devoted to one target course, the three courses being repeated in the order (i), (ii), (iii) three times. In each block the five control laws were taken in random order, three targets being tracked with each. The first of these three runs was not included in the analysis. However, inspection of the results showed that, in general, the discarded runs differed very little from the retained runs, and that there was no apparent tendency for performance to improve from the second to the third run.

A.7. In Experiment 2 each subject undertook 5 blocks of 27 runs each. Each block was devoted to one of the five control laws, these being arranged in a different random order for each subject. Three introductory runs (discarded for the purposes of analysis) were given at the start of each block, followed by eight runs of each of the three target courses, these 24 being undertaken in random order. There was no apparent tendency for accuracy to improve within any one block, although performance did seem to become more variable towards the end of the block.

A.8. It was evident that, if the experimental points chosen were regarded as equidistant on a velocity vs acceleration scale, a quadratic relationship would be quite inadequate to describe the data. The following scales were therefore devised:

(a) $x_2 = A/V$

(b) $x_2 = A/(V + 0.5A)$

and (c) $x_2 = A/(V + A)$

Using these scales, quadratic relationships were fitted to the data, taking each subject, experiment and target course separately. Scale (a) gave the best fit in five cases; scale (b) in two; and scale (c) in five. Taken overall, scale (b) seemed to give the best results, as even when it was intermediate between the other two, its fit was little inferior to the best.

A.9. The straight flying targets resembled each other in that their curvature was quite pronounced, and that their optima appeared to be similarly placed. For target course (i) calculated optimal acceleration terms were 3.00, 3.00, 2.96 and 2.86 deg/sec^2; the figures for target course (ii) being 2.20, 2.86, 2.70 and 2.83 deg/sec^2. Averaging over these two target courses thus yields an optimum at:

velocity	4.40 deg/sec
acceleration	2.80 deg/sec^2

A.10. For the weaving target the curvature in this dimension was far less pronounced. Even so, the data yielded remarkably consistent optima, the relevant acceleration terms being 3.27, 3.29, 3.28 and 3.22 deg/sec^2. Averaging once again, we have an optimim at:

velocity	3.46 deg/sec
acceleration	3.27 deg/sec^2

Experiment 3

A.11. For this experiment the velocity and acceleration terms were held constant at 4.4deg/sec. and 2.8 deg/sec^2, respectively. The following values of position element were used:

(a) 0.5

(b) 1.0

(c) 1.5

(d) 2.0

(e) 2.5 deg.

Target courses were the same as most in Experiments 1 and 2.

A.12. Each subject undertook 6 blocks of 15 runs each, each block being devoted to one target course, in similar fashion to Experiment 1. Each subject and each target course was analysed separately. If the position term was expressed on the scale $x_1 = \log P$, an adequate fit was obtained by a quadratic equation in all but one case. The optima were rather scattered, however, their values being 1.38, 1.80, (2.80), 1.77, 2.00 and 1.26 deg. (the bracketed value being that yielded by the case with inadequate fit). The mean of these values is 1.84 deg. (1.64 deg, if the questionable value is omitted).

Experiment 4

A.13 The final experiment of this series employed three levels of position (0.8, 1.6 and 3.2 deg.) and four levels of velocity-acceleration (5.2:2.4, 4.4:2.8, 3.6:3.2 and 2.8:3.6 (deg/sec. : deg/sec^2). The two target courses had crossing distances of:

(i) 10,000 ft

(ii) 2,500 ft, with an added 3g vertical weave.

Each block of 24 runs was confined to one target course, and consisted of two tracking runs for each of the 12 control laws, undertaken in random order. Five blocks were undertaken with target course (i) and four blocks with target course (ii).

A.14. The two target courses were separated for the purposes of analysis. Three separate analyses were carried out for each target course, defining the position scale always as $x_1 = \log P$, but using the three alternative definitions for the velocity-acceleration dimension:

(a) $x_2 = A/V$

(b) $x_2 = A/(V + 0.5A)$

(c) $x_2 = A/(V + A)$

The scale as defined by (b) gave the best fit for target course (i) and as defined by (a) for target course (ii), although the residual sum of squares varied but little for different scales within each target course, and all were well within the limits of experimental error. Most important, for target course (i) the three scales yielded very similar optima as follows:

Velocity-Acceleration Scale Defined by	Fitted Optima		
	Position	Velocity	Acceleration
	deg.	deg/sec.	deg/sec^2
(a) $x_2 = A/V$	1.86	4.64	2.68
(b) $x_2 = A/(V + 0.5A)$	1.84	4.60	2.70
(c) $x_2 = A/(V + A)$	1.83	4.62	2.69

A.15. Target course (ii) appeared to result in a ridge system, with no true optimum. Definition (b) was therefore accepted for the velocity-acceleration dimension. With x_1 and x_2 thus defined, the final equations were:

for target course (i):

$$\hat{y} = 0.7943 - 0.9787x_1 - 1.5165x_2 + 0.8156x_1^2 + 1.3176x_2^2 + 1.2062x_1x_2$$

and for target course (ii):

$$\hat{y} = 0.7167 - 0.1863x_1 + 0.0125x_2 + 0.6227x_1^2 + 0.0015x_2^2 - 0.2380x_1x_2$$

These relationships are illustrated in Figures A.1 and A.2, where the experimental points, together with the measured mean log r.m.s. errors, are also shown.

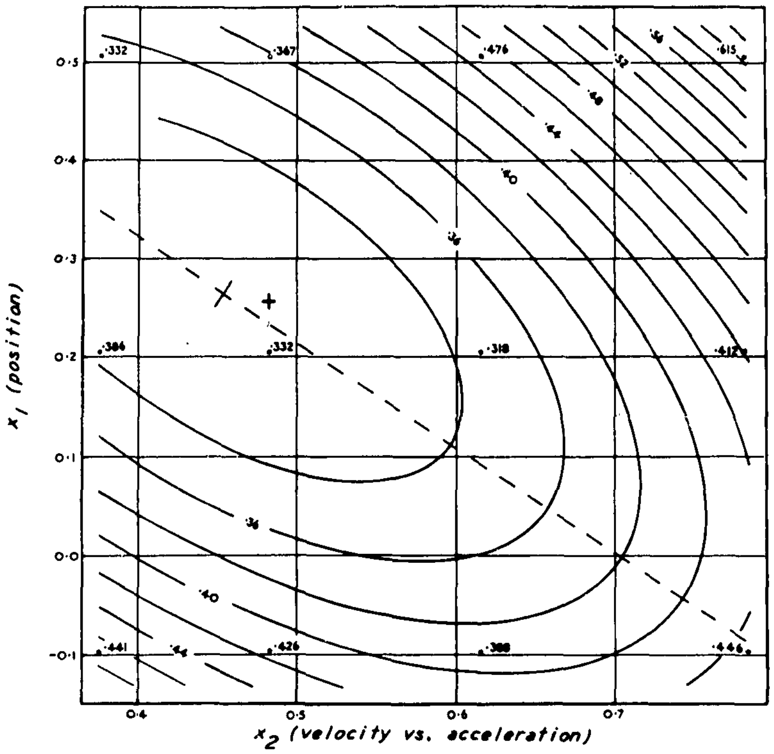

Figure A1 Tracking accuracy. Target crossing distance 10,000 ft. Contours are spaced at intervals of 0.02 log r.m.s. error. The point chosen as the centre for the main experiment design is marked by a vertical cross.

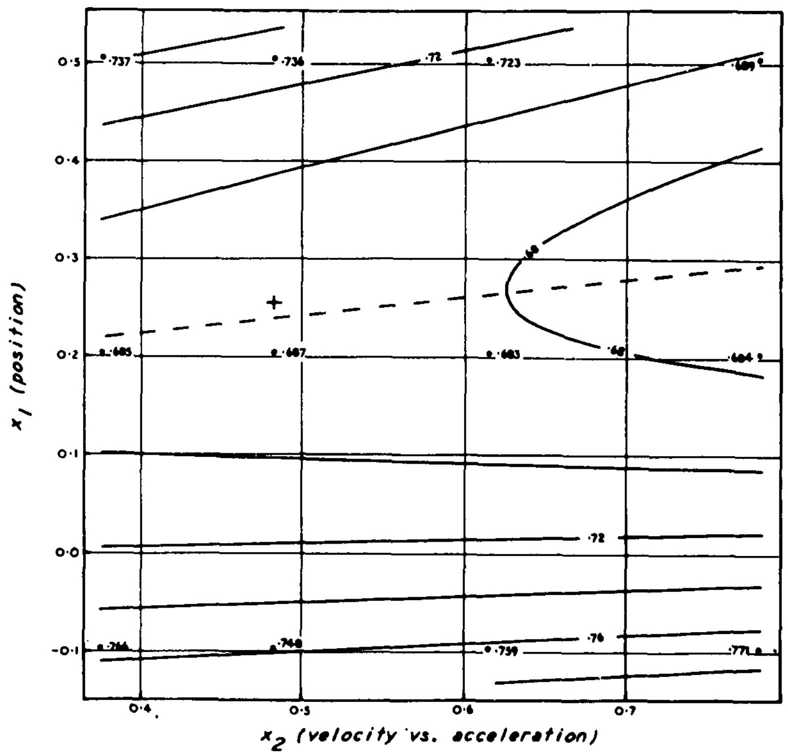

Figure A2 Tracking accuracy. Target crossing distance 2,500 ft. with 3g vertical weave. Contours are spaced at intervals of 0.02 log r.m.s. error. The point chosen as the centre for the main experiment design is marked by a vertical cross.

Discussion

A.16. By and large, results from this series of investigations were mutually consistent. On the face of it, they appear to form a reasonable basis for planning a full-scale experiment. As a centre point for the design of the latter, the location P = 1.8 deg, V = 4.4 deg, A = 2.8 deg/sec^2. ($x_1 = 0.2553$, $x_2 = 0.4828$) would seem to be satisfactory.

A.17. From the evidence of this series, it was concluded that suitable scales for expressing control law parameters are given by:

$$x_1 = \log_{10} P$$

$$x_2 = A/(V + 0.5A), \quad (V + 2A = 10)$$

where

P is the value of the position term in deg.

V is the value of the velocity term in deg/sec.

A is the value of the acceleration term in deg/sec^2.

A.18. There was significant learning even during the course of Experiment 4, and so it was assumed that the design of the main experiment must allow for this effect. It appeared, also, that over-large blocks of tracking runs were to be avoided if possible. On the other hand, subjects seemed to adapt to a new control law extremely quickly, although the possibility of long-term transfer of training effects cannot be ruled out. In the short term, however, there would seem to be little need to provide lengthy adapting periods when switching to a new law.

A.19. Finally, it was noticed in these experiments that settling time at the start of a run was greatly affected by control law. If the initial non-scored period in each run is made very brief, this could obviously have a significant effect on final integrated r.m.s. error. It was decided, therefore, that the period allowed for initial acquisition and settling should be increased to 9 sec. in the main experiment, and that acquisition performance and tracking accuracy should be assessed separately.

References

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ANNEX B

EXPERIMENTAL DESIGN AND ANALYSIS

Experimental Design

B.1. In designing the main experiment we have considered the following effects:

(S) operator-to-operator variation

(A) order (or learning) effect

(B) control law parameters

(C) target course.

(Not included in the above list is the transfer of training effect, which we have sought to reduce to negligible proportions by ignoring results obtained on the first run with each new control law.) The factors S, A and B have been arranged in a Latin Square design, all measurements taking place at all levels of factor C. It has been assumed that there is no appreciable interaction between factor A (order) and factors S and B. No such assumption, however, has been made for the SB (subjects-by-control law) interaction. As a result it will be noted that the main effect of factor A will be confounded with the SB interaction, although the main effects of factors S and B (which are our chief concern in this study) will not be confounded.

B.2. The control law is itself varied in two dimensions and the proper placing of experimental points in this plane requires some consideration. Our main aim is, of course, to approximate a functional relationship between the response (tracking accuracy or acquisition time) and control law parameters by means of a second-order polynomial. To minimize bias due to neglected terms up to and including the fourth order, a so-called "rotatable" design is required. Unlike the factorial design used in Experiment 4 of Annex A, a rotatable design will generate information such that the response is estimated with constant variance at all points equidistant from its origin. This is a useful property, as little will be known in advance about the orientation of any chosen design relative to the response surface. These points are considered in some detail in References B.2 and B.3.

B.3. The class of second-order rotatable designs we have considered consist of a ring of at least five equi-spaced points, plus one or more points at the centre of this circle. In Reference B.2 it is shown that when we have reason to suspect the accuracy of our model, it is safest to concentrate our points on the circumference of the circle rather than at its centre, and to restrict the scope of the design. Accordingly, with the number of subjects, and thus the number of points, fixed at 11, we have placed eight points on the circumference of the design and three at its centre. Working in terms of the design variables v_1 and v_2, obtained from the control law parameters by the simple linear transformations

$$v_1 = (x_1 - 0.255273)/0.176091$$

$$v_2 = (x_2 - 0.482759)/0.107759$$

the design matrix ($\underline{v}$) and the derived matrix of independent variables ($\underline{V}$) are given in Table B.I. (The same design matrix formation is given in terms of the untransformed variables in Table I of the main text.) Given the measured response, y_i, at each design point, the estimate $\underline{b}$ of the vector β is given by

$$\underline{b} = (\underline{V}'\underline{V})^{-1}\ \underline{V}'\underline{y}.$$

TABLE B.I

EXPERIMENTAL DESIGN

Experimental Point	Design Matrix		Matrix of Independent Variables					
	v_1	v_2	v_0	v_1	v_2	v_1^2	v_2^2	v_1v_2
1	0	0	1	0	0	0	0	0
2	0	0	1	0	0	0	0	0
3	0	0	1	0	0	0	0	0
4	-1	-1	1	-1	-1	1	1	1
5	-1	1	1	-1	1	1	1	-1
6	1	-1	1	1	-1	1	1	-1
7	1	1	1	1	1	1	1	1
8	$-\sqrt{2}$	0	1	$-\sqrt{2}$	0	2	0	0
9	$\sqrt{2}$	0	1	$\sqrt{2}$	0	2	0	0
10	0	$-\sqrt{2}$	1	0	$-\sqrt{2}$	0	2	0
11	0	$-\sqrt{2}$	1	0	$\sqrt{2}$	0	2	0

Analysis of Variance

B.4. Both for tracking accuracy and for acquisition time the interaction between control law and target course proved significant well beyond the 0.0001 level. Target courses have therefore been considered separately, and analyses of variance are shown in Tables B.II to B.VII. It will be noted that the sums of squares due to control law have been subdivided into a portion due to regression and a residual. In no case did this residual prove significantly different from the subject-by-control law interaction, and so these two terms have been pooled to form a single error term. It is worth noting from the results that the subject-by-control law interaction term reached the 0.05 level of significance in only one of the six analyses.

B.5. The fitted regression coefficients (quoted in a form appropriate to x_1 and x_2 rather than v_1 and v_2) are given in Table II of the main text.

TABLE B.II

ANALYSIS OF VARIANCE, TRACKING ACCURACY
10,000 ft. Crossing Distance

Source	SS	df	MS	F	Significance
S Subjects	1.0732	10	0.10732		
A Order	0.2007	10	0.02007		
B Control Law	(0.1855	10)			
Regression	0.1274	5	0.02548	3.19	$p < .025$
Residual	0.0581	5)			
			0.00799		
SB	0.7008	90)		1.05	N.S.
W Within Cells	0.9015	121	0.00745		

TABLE B.III

ANALYSIS OF VARIANCE, TRACKING ACCURACY
2,500 ft. Crossing Distance

Source	SS	df	MS	F	Significance
S Subjects	2.4173	10	0.24173		
A Order	0.0593	10	0.00593		
B Control Law	(0.2111	10)			
Regression	0.1739	5	0.03478	4.49	$p < .005$
Residual	0.0372	5)			
			0.00775		
SB	0.6990	90)		1.32	N.S.
W Within Cells	0.7108	121	0.00588		

TABLE B.IV

ANALYSIS OF VARIANCE, TRACKING ACCURACY
5,000 ft. Crossing Distance, Plus 2g Weave

Source	SS	df	MS	F	Significance
S Subjects	1.9034	10	0.19034		
A Order	0.0681	10	0.00681		
B Control Law	(1.2824	10)			
Regression	1.2766	5	0.25533	57.96	$p < .0001$
Residual	0.0057	5)			
			0.00441		
SB	0.4127	90)		1.54	$p < .05$
W Within Cells	0.3613	121	0.00299		

TABLE B.V

ANALYSIS OF VARIANCE, ACQUISITION TIME
10,000 ft. Crossing Distance

Source	SS	df	MS	F	Significance
S Subjects	54.538	10	5.4538		
A Order	3.958	10	0.3958		
B Control Law	(145.270	10)			
Regression	140.835	5	28.1670	34.36	p < .0001
Residual	4.435	5	0.8197		
SB	73.435	90		1.24	N.S.
W Within Cells	79.680	121	0.6585		

TABLE B.VI

ANALYSIS OF VARIANCE, ACQUISITION TIME
2,500 ft. Crossing Distance

Source	SS	df	MS	F	Significance
S Subjects	31.347	10	3.1347		
A Order	4.716	10	0.4716		
B Control Law	(16.555	10)			
Regression	13.359	5	2.6719	4.20	p < .0001
Residual	3.195	5	0.6369		
SB	57.308	90		1.23	N.S.
W Within Cells	62.586	121	0.5172		

TABLE B.VII

ANALYSIS OF VARIANCE, ACQUISITION TIME
5,000 ft. Crossing Distance, Plus 2g Weave

Source	SS	df	MS	F	Significance
S Subjects	97.717	10	9.7717		
A Order	19.154	10	1.9154		
B Control Law	(111.928	10)			
Regression	106.179	5	21.2358	12.69	p < .0001
Residual	5.749	5	1.6732		
SB	153.204	90		1.29	N.S.
W Within Cells	159.441	121	1.3177		

Canonical Analysis of Fitted Equations

B.6. Some insight into the nature of the fitted equations can be obtained by reducing them to their canonical form. In essence, this consists of shifting the origin to the centre of the system of contour curves, and rotating the co-ordinate axes to correspond to the principal axes of the system. The quadratic equations then reduce to the form:

$$y - y_o = B_{11}\, w_1^{\,2} + B_{22}\, w_2^{\,2}$$

where

y_o is the predicted response at the centre of the system

and w_1 and w_2 are the new co-ordinate axes.

It is possible to test whether B_{22} differs significantly from zero, and thus it is possible to judge whether all the curvature of the response surface effectively takes place in one plane, so that the system can be approximated by a surface containing a stationary line. From Reference B.3. Section 10, the significance of B_{22} can be established by referring the quantity

$$\frac{4\, NB_{22}^{\,2}}{2(\lambda^{-1} + 1)\, c^2 s^2} = \frac{hB_{22}^{\,2}}{s^2}$$

to tables of the F distribution with degrees of freedom (2, ϕ). s^2 is an estimate of variance having ϕ degrees of freedom, derived in our case from the relevant analysis of variance table, with $\phi = 95$. The other symbols are defined in the reference quoted above. For our design $N = 11$, $\lambda = 11/16$, $1 = 8/3$ and $c = 11/8$; and so $h = 2.8235$. Table B.VIII lists the computed values of B_{11} and B_{22}, together with the statistic defined above. As $F_{0.25}(2,95) = 1.41$, none of the B_{22} coefficients prove significant at the 0.25 level.

TABLE B.VIII

COEFFICIENTS OF CANONICAL EQUATIONS

Target Course (crossing distance)	B_{11}	B_{22}	s^2	$hB_{22}^{\,2}/s^2$
Tracking Accuracy				
10,000 ft.	0.027435	-0.004624	0.0003631	0.166
2,500 ft.	0.017573	-0.009276	0.0003523	0.622
5,000 ft. + weave	0.024611	-0.000806	0.0002002	0.001
Acquisition Time				
10,000 ft.	0.38892	-0.03748	0.03726	0.107
2,500 ft.	0.17511	-0.01346	0.02895	0.177
5,000 ft. + weave	0.24934	-0.12547	0.07605	0.584

B.7. The equations of the lines forming the canonical axes w_1 and w_2 are given by:

$$\begin{bmatrix} w_1 \\ \\ w_2 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} v_1 - v_1^{o} \\ \\ v_2 - v_2^{o} \end{bmatrix}$$

where

$\underline{M}$ is the orthogonal transformation matrix

and v_1^{o} and v_2^{o} are the origins of the system of contours.

The computed elements of $\underline{M}$ and the values of v_1^{o} and v_2^{o} are given in Table B.IX. Now in a sense it is true that the principal axes are the lines along which there is respectively least and most variation of response. But by "variation" is meant variation per unit distance, and it must be borne in mind that the scales v_1 and v_2 are entirely arbitrary, with units which are not comparable. A simple change of scale (from v_1 and v_2 to x_1 and x_2, for instance) would bring about a different orientation of the principal axes. The canonical axes found here and illustrated in Figures 1 to 7 (main text) and A.1 to A.2 are those obtained using the experimental scales v_1 and v_2, and the limitations in their interpretation must be borne in mind.

TABLE B.IX

ELEMENTS OF ORTHOGONAL TRANSFORMATION MATRICES AND ORIGINS OF CANONICAL SYSTEMS

Target Course (crossing distance)	m_{11}	m_{12}	m_{21}	m_{22}	v_1^{o}	v_2^{o}
Tracking Accuracy						
10,000 ft.	0.9037	-0.4282	0.4282	0.9037	0.3497	0.4699
2,500 ft.	0.9867	0.1625	0.1625	-0.9867	-0.5985	- 0.4953
5,000 ft. + weave	0.9900	0.1408	0.1408	-0.9900	-0.1016	-10.8987
Acquisition Time						
10,000 ft.	0.9861	0.1662	0.1662	-0.9861	-1.7820	4.9769
2,500 ft.	0.9895	-0.1446	0.1446	0.9895	0.6748	7.0287
5,000 ft. + weave	0.8860	0.4637	0.4637	-0.8860	-1.5557	- 0.0956

Confidence Intervals

B.8. The width of the $1 - \alpha$ confidence interval for the predicted response at a given point (v_1^{o}, v_2^{o}) will be given by:

$$2t_{1-\alpha/2} \{ s^2(\underline{v}^{o})'(\underline{V}'\underline{V})^{-1}\underline{v}^{o} \}^{\frac{1}{2}} = 2t_{1-\alpha/2}\, s$$

where s^2 is an estimate of variance based on ϕ degrees of freedom

and $t_{1-\alpha/2}$ is Student's t at the appropriate percentage point.

For the design used, $\kappa = 5/8$ at any location on the outer circle of points. The width of the 50% and 90% confidence intervals for y at such a location are given in Table III of the main text.

Variance Due to Operators

B.9. It is important to know whether there is the same spread of operator tracking ability (measured in log units) for all the target courses in this experiment. In fact, the hypothesis we wish to test is that the variance due to operators is the same for each target, and that all covariances are equal. The matrix of observed variances and covariances of tracking score totals is given in Table B.X. Applying Box's test of the above hypothesis(B.1), we find $F(4, 2400) = 1.97$, which fails to attain significance at the 0.05 level.

TABLE B.X

COVARIANCE MATRIX
OPERATOR TRACKING ERRORS

Target Course (crossing distance)	(i)	(ii)	(iii)
(i) 10,000 ft.	2.3611	3.1892	2.2912
(ii) 2,500 ft.	3.1892	5.3180	4.0737
(iii) 5,000 ft. + weave	2.2912	4.0737	4.1874

B.10. In the statistical model for the analysis of variance based on the data for all target courses, the expected value for the mean square for subjects is given by:

$$E(MS_s) = \sigma_e^2 + 66\,\sigma_s^2$$

and that for the within cells mean square by:

$$E(MS_w) = \sigma_e^2$$

where σ_e^2 is the variance due to error

and σ_s^2 is the variance due to operator differences in the population.

Our estimate of the variance due to operator differences is thus:

$$\begin{aligned} s^2 &= (MS_s - MS_w)/66 \\ &= (0.469311 - 0.005437)/66 \\ &= 0.007028 \end{aligned}$$

and so $s = 0.08383$

and a range $\pm$ 1.6449 s gives 0.2768, equivalent to a factor of 1.9. Using the approximate method of Satterthwaite[B.4], the 50% confidence interval is given by $0.0740 < \sigma < 0.1001$ and the 90% interval by $0.0579 < \sigma < 0.1320$.

Relationships Between Accuracy and Other Variables

B.11. The linear regression equation describing the relationship between individual mean log r.m.s. tracking errors ($\hat{y}$) and mean acquisition time (x_3) is:

$$\hat{y} = 0.35240 + 0.00230x_3$$

It will be noted that the relationship is positive, an increase in tracking error being associated with an increase in acquisition time. From the analysis of variance (Table B.XI) it will be seen that the relationship is statistically significant, and that the product-moment correlation coefficient between the two indices is 0.878.

TABLE B.XI

REGRESSION ANALYSIS
TRACKING ACCURACY vs ACQUISITION TIME

Source	SS	df	MS	F	Significance
Regression	239.006	1	239.006	30.41	$p < .0001$
Residual	70.742	9			
Total	309.748	10			

B.12. For tracking accuracy and TSG score (x_4) the regression equation is:

$$\hat{y} = 0.65723 + 0.0058x_4$$

This represents an expected increase in r.m.s. tracking error of approximately 1.3% per point increase in TSG. This compares with a figure of 5% found in the experiments reported in Reference B.5. From the analysis of variance shown in Table B.XII it will be seen that the relationship obtained here fails to attain statistical significance.

TABLE B.XII

REGRESSION ANALYSIS
TRACKING ACCURACY vs TSG SCORE

Source	SS	df	MS	F	Significance
Regression	31.059	1	31.059	1.00	N.S.
Residual	278.689	9			
Total	309.748	10			

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B.5. Speight, L.R. The effect of type of control on tracking accuracy in the simulated ET 316 system. AORE Report No. 1/65, 1965 (SECRET).

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